and events, Probability axioms and properties, Conditional probability, Bayes' theorem, and independent events; Discrete random variables & probability distributions, Expected values; Probability distributions: Binomial, geometric, hypergeometric, negative binomial, Poisson, and Poisson distribution as a limit.

UNIT-II: Continuous Probability Distributions **(15 hours)**
Continuous random variables, Probability density functions, Uniform distribution, Cumulative distribution functions and expected values, The normal, exponential, and lognormal distributions.

UNIT-III: Central Limit Theorem and Regression Analysis **(15 hours)**
Sampling distribution and standard error of the sample mean, Central Limit Theorem, and applications; Scatterplot of bivariate data, Regression line using principle of least squares, Estimation using the regression lines; Sample correlation coefficient and properties.

Practical (30 hours): Software labs using Microsoft Excel or any other spreadsheet.

1) Presentation and analysis of data (univariate and bivariate) by frequency tables, descriptive statistics, stem-and-leaf plots, dotplots, histograms, boxplots, comparative boxplots, and probability plots ([1] Section 4.6).
2) Fitting of binomial, Poisson, and normal distributions.
3) Illustrating the Central Limit Theorem through Excel.
4) Fitting of regression line using the principle of least squares.
5) Computation of sample correlation coefficient.

Essential Reading

1. Devore, Jay L. (2016). Probability and Statistics for Engineering and the Sciences (9th ed.). Cengage Learning India Private Limited. Delhi. Indian Reprint 2022.

Suggestive Reading

- Mood, A. M., Graybill, F. A., & Boes, D. C. (1974). Introduction to the Theory of Statistics (3rd ed.). Tata McGraw-Hill Pub. Co. Ltd. Reprinted 2017.

DSE Courses of B.A. (Prog.) Semester-VI
Category-II

DISCIPLINE SPECIFIC ELECTIVE COURSE – 2(i): DISCRETE DYNAMICAL SYSTEMS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		

Discrete Dynamical Systems	**4**	**3**	**0**	**1**	**Class XII pass with Mathematics**	**NIL**

Learning Objectives: The primary objective of this course is to introduce:

- The fundamental concepts of discrete dynamical systems and emphasis on its study through several applications.
- The concepts of the fixed points, chaos and Lyapunov exponents for linear and nonlinear equations have been explained through examples.
- Various applications of chaos in higher dimensional models.

Learning Outcomes: This course will enable the students to:

- Understand the basic concepts of difference equation, chaos and Lyapunov exponents.
- Obtain fixed points and discuss the stability of the dynamical system.
- Find Lyapunov exponents, Bifurcation, and Period-doubling for nonlinear equations.
- Analyze the behavior of different realistic systems with chaos cascade.

SYLLABUS OF DSE-2(i)

UNIT-I: Discrete-time Models (12 hours)

Dynamical systems concepts and examples; Some linear models: Bouncing ball, investment growth, population growth, financial, economic and linear price models; Nonlinear models: Density-dependent population, contagious-disease, economic and nonlinear price models; Some linear systems models: Prey-predator, competing species, overlapping-generations, and economic systems.

UNIT-II: Linear Equations, Systems, their Solutions and Dynamics (18 hours)

Autonomous, non-autonomous linear equations and their solutions, time series graphs; Homogenous, non-homogeneous equations and their solutions with applications; Dynamics of autonomous linear equations, fixed points, stability, and oscillation; Homogeneous, non-homogeneous linear systems and their dynamics, solution space graphs, fixed points, sinks, sources and saddles.

UNIT-III: Nonlinear Equations, their Dynamics and Chaos (15 hours)

Autonomous nonlinear equations and their dynamics: Exact solutions, fixed points, stability; Cobweb graphs and dynamics: Linearization; Periodic points and cycles: 2-cycles, m-cycles, and their stability; Parameterized families; Bifurcation of fixed points and period-doubling; Characterizations and indicators of chaos.

Practical (30 hours)- Use of Excel/SageMath/MATHEMATICA/MATLAB/Scilab Software:

1. If Rs. 200 is deposited every 2 weeks into an account paying 6.5% annual interest compounded bi-weekly with an initial zero balance:
   (a) How long will it take before Rs. 10,000/- is in account?
   (b) During this time how much is deposited and how much comes from interest?

(c) Create a time series graph for the bi-weekly account balances for the first 40 weeks of saving scenario.

[1] Computer Project 2.5 pp. 68

2. (a) How much can be borrowed at an annual interest rate of 6% paid quarterly for 5 years in order to have the payments equal Rs. 1000/- every 3 months.

   (b) What is the unpaid balance on this loan after 4 years.

   (c) Create a time series graph for the unpaid balances each quarter for the loan process.

   [1] Computer Project 2.5 pp. 68

3. Four distinct types of dynamics for any autonomous linear equation:

   $x_{n+1} = a\,x_n + b$ for different values of a and b.

   [1] Dynamics of autonomous linear equation, pp. 74

4. Find all fixed points and determine their stability by generating at least the first 100 iterates for various choices of initial values and observing the dynamics

   a. $I_{n+1} = I_n - r\,I_n + s\,I_n\,(1 - I_n\,10^{-6})$
   for: (i) $r = 0.5, s = 0.25$, (ii) $r = 0.5, s = 1.75$, (iii) $r = 0.5, s = 2.0$.

   b. $P_{n+1} = \frac{1}{P_n} + 0.75\,P_n + c$
   for: $(i)\ c = 0$; $(ii)\ c = -1$; $(iii)\ c = -1.25$; $(iv)\ c = -1.38$.

   c. $x_{n+1} = a\,x_n\,(1 - x_n^2)$
   for: $(i)\ a = 0.5$; $(ii)\ a = 1.5$; $(iii)\ a = 2.25$; $(iv)\ a = 2.3$.

   [1] Computer Project 3.2 pp. 110

5. Determine numerically whether a stable cycle exists for the given parameter values, and if so, its period. Perform at least 200 iterations each time and if a cycle is found (approximately), use the product of derivatives to verify its stability.

   a. $P_{n+1} = r\,P_n\left(1 - \frac{P_n}{5000}\right)$, for: $(i)\ r = 3.4$; $(ii)\ r = 3.5$;
   $(iii)\ r = 3.566$; $(iv)\ r = 3.569$; $(v)\ r = 3.845$.

   b. $P_{n+1} = r\,P_n\,e^{-P_n/1000}$
   for: $(i)\ r = 5$; $(ii)\ r = 10$; $(iii)\ r = 14$; $(iv)\ r = 14.5$; $(v)\ r = 14.75$.

   [1] Computer Project 3.5 pp. 154

6. Find through numerical experimentation the approximate intervals of stability of the (a) 2-cycle; (b) 4-cycle; (c) 8-cycle; (d) 16-cycle; (e) 32-cycle for the following

   a. $f_r(x) = r\,x\,e^{-x}$
   b. $f_r(x) = r\,x^2\,(1 - x)$
   c. $f_a(x) = x\,(a - x^2)$
   d. $f_c(x) = \frac{2}{x} + 0.75\,x - c$

   [1] Computer Project 3.6 pp. 164

7. Through numerical simulation, show that each of the following functions undergoes a period doubling cascade: (**[1] Computer Project 3.7 pp.175**)

   a. $f_r(x) = r\,x\,e^{-x}$
   b. $f_r(x) = r\,x^2\,(1 - x)$
   c. $f_r(x) = r\,x\,e^{-x^2}$
   d. $f_r(x) = \frac{r\,x}{(x^2+1)^2}$
   e. $f_a(x) = x\,(a - x^2)$

8. Discuss (a) Pick two initial points close together, i.e., that perhaps differ by 0.001 or 0.00001, and perform at least 100 iterations of $x_{n+1} = f(x_n)$. Do solutions exhibit sensitive dependence on initial conditions?

   (b) For several random choices of x_0 compute at least 1000 iterates x_n and draw a frequency distribution using at least 50 sub-intervals. Do dense orbits appear to exit?

   (c) Estimate the Lyapunov exponent L by picking several random choices of x_0 and computing $\frac{1}{N}\sum_{n=1}^{N} \ln|f'(x_n)|$ for $N = 1000, 2500, 5000, etc.$

   Does L appear to be positive? i). $f(x) = 2 - x^2$ ii). $f(x) = \frac{2}{x} + \frac{3x}{4} - 2$.

   [1] Computer Project 3.8 pp. 187
9. Show that $f(x) = r\,x\,(1-x)$ for $r > 4$ and $f(x) = 6.75\,x^2\,(1-x)$ have horseshoes and homoclinic orbits, and hence chaos. **[1] Computer Project 3.8 pp. 188**
10. Find the fixed point and determine whether it is a sink, source or saddle by iterating and graphing in solution space the first few iterates for several choices of initial conditions. **[1] Computer Project 4.2 pp. 207**

    a. $x_{n+1} = x_n - y_n + 30$

       $y_{n+1} = x_n + y_n - 20.$

    b. $x_{n+1} = x_n + y_n$

       $y_{n+1} = x_n - y_n.$

Essential Reading

1. Marotto, Frederick R. (2006). Introduction to Mathematical Modeling Using Discrete Dynamical Systems. Thomson, Brooks/Cole.

Suggestive Readings

- Devaney, Robert L. (2022). An Introduction to Chaotic Dynamical Systems (3rd ed.). CRC Press, Taylor & Francis Group, LLC.
- Lynch, Stephen (2017). Dynamical Systems with Applications using Mathematica® (2nd ed.). Birkhäuser.
- Martelli, Mario (1999). Introduction to Discrete Dynamical Systems and Chaos. John Wiley & Sons, Inc., New York.

DISCIPLINE SPECIFIC ELECTIVE COURSE – 2(ii): INTRODUCTION TO MATHEMATICAL MODELING

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course (if any)
		Lecture	Tutorial	Practical/ Practice		